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EFFECT OF FEEDPOINT LOCATION FOR A BINARY MIXTURE ON THE EFFICIENCY OF SEPARATION IN A THERMAL-DIFFUSION COLUMN

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We show that there exists an optimal position of the feedpoint of a column at which the separation efficiency is maximum and for which, at the same time, the energy consumption is a minimum.

Rozen [1] was the first to solve the problem of the effect produced by the position of the feedpoint of the initial binary mixture on the separation efficiency in column apparatuses. However, the method used for this was worked out in terms of the theory of rectification, which had no real physical analogs in the theory of thermal-diffusion equipment. (For example, finding the points of intersection of the operating lines in the liquid—vapor equilibrium diagram.)

In connection with the development of a technology for the thermal-diffusion separation of petroleum products, it became necessary to solve the above-mentioned problem of finding the optimum coordinate of the feedpoint of the mixture being separated for a thermal-diffusion column, shown schematically in Fig. 1. The upper part of the column (I) is the concentrating part, while the lower is the stripping part (II). The input to the column is at the cross section whose dimensionless coordinate is y_0. For each part of the column, in accordance with [2], we can write the transfer equation in the form

$$\tau' = H\left[c'(1-c') - \frac{dc'}{dy} + \varkappa_e c'\right],$$

$$\tau'' = H\left[c''(1-c'') - \frac{dc''}{dy} - \varkappa_i c''\right].$$

The simultaneous solution of this system of equations leads, since it is nonlinear, to results which are cumbersome and inconvenient for analysis [3], and for this reason it is desirable to resort to linearization of the quadratic term in the form

$$c(1-c) = a + bc, \tag{1}$$

which is entirely applicable to the solution of some separation problems. Then, taking account of (1), the above system takes the form

$$\begin{aligned} \tau' &= H\left[a + (b + \varkappa_e) c' - \frac{dc'}{dy}\right], \\ \tau'' &= H\left[a + (b - \varkappa_i) c'' - \frac{dc''}{dy}\right]. \end{aligned} \tag{2}$$

Since we are considering the stationary state, we know that $d\tau/dy = 0$, and from (2) we obtain the system

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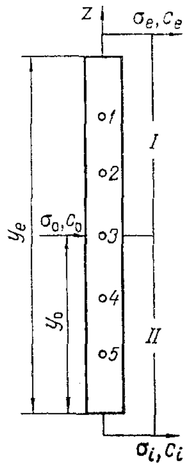

Fig. 1. Scheme of a thermal-diffusion column with input at the cross section y_0. The circles indicate the feedpoints of the mixture being separated in the experiment.

$$\frac{d^2c'}{dy^2} - (b+\varkappa_e)\frac{dc'}{dy} = 0, \qquad \frac{d^2c''}{dy} - (b-\varkappa_i)\frac{dc''}{dy} = 0, \tag{3}$$

whose solution must satisfy the boundary conditions

$$\tau'|_{y=y_e} = \sigma_e c_e,\ \tau''|_{y=0} = -\sigma_i c_i'\ ,c'|_{y=y_0} = c''|_{y=y_0} \tag{4}$$

and the condition of conservation of the target component

$$\sigma_i c''|_{y=0} + \sigma_e c'|_{y=y_e} = \sigma_0 c_0. \tag{5}$$

Then the solution of system (3) yields

$$\begin{aligned} c' = \frac{1}{\psi_1}\Big\{&\varkappa_0\varkappa_e c_0\{\varkappa_i - b\exp[(b-\varkappa_i)y_0]\} - a(\varkappa_i - b)\times \\ &\times\{\varkappa_i\exp[-(b+\varkappa_e)(y_e-y_0)] + \varkappa_e\exp[(b-\varkappa_i)y_0]\} + \\ &+\varkappa_0(a+bc_0)\exp[-(b+\varkappa_e)(y_e-y)]\{\varkappa_i - b\exp[(b-\varkappa_i)y_0]\}\Big\}, \end{aligned} \tag{6}$$

$$\begin{aligned} c'' = \frac{1}{\psi_1}\Big\{&\varkappa_0\varkappa_i c_0\{\varkappa_e + b\exp[-(b+\varkappa_e)(y_e-y_0)]\} + \\ &+a(\varkappa_e+b)\{\varkappa_i\exp[-(b+\varkappa_e)(y_e-y_0)] + \varkappa_e\exp[(b-\varkappa_i)y_0]\} - \\ &-\varkappa_0(a+bc_0)\exp[(b-\varkappa_i)y]\{\varkappa_e + b\exp[-(b+\varkappa_e)(y_e-y_0)]\}\Big\}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} \psi_1 = {}&\varkappa_i(\varkappa_i - b)\{\varkappa_e + b\exp[-(b+\varkappa_e)(y_e-y_0)]\} + \\ &+\varkappa_e(\varkappa_e+b)\{\varkappa_i - b\exp[(b-\varkappa_i)y_0]\}, \end{aligned} \tag{8}$$

and $\varkappa_0 = \varkappa_e + \varkappa_i$.

We consider some special cases:

1. Concentration of the Target Component is Low, i.e., c << 1. Then, in accordance with (1), $a = 0$, b = 1, and instead of (6), (7), we obtain

$$c' = \frac{\varkappa_0 c_0}{\psi_1}\{\varkappa_1 - \exp[(1-\varkappa_i)y_0]\}\{\varkappa_e + \exp[-(1+\varkappa_e)(y_e-y)]\}, \tag{9}$$

$$c'' = \frac{\varkappa_0 c_0}{\psi_1}\{\varkappa_1 - \exp[(1-\varkappa_i)y]\}\{\varkappa_e + \exp[-(1+\varkappa_e)(y_e-y_0)]\}, \tag{10}$$

hence the degree of separation in the concentrating part of the column, i.e., when $c'|_{y=y_e} = c_e$, will be

$$\frac{c_e}{c_0} \equiv q_e = \frac{\varkappa_0}{\psi_1}(\varkappa_e + 1)\{\varkappa_i - \exp[(1-\varkappa_i)\,y_0]\}, \tag{11}$$

and the degree of separation in the stripping part ($c''|_{y=0} = c_i$) will be

$$\frac{c_0}{c_i} \equiv q_i = \frac{\psi_1}{\varkappa_0}\left\{(\varkappa_i - 1)\{\varkappa_e + \exp[-(1+\varkappa_e)(y_e - y_0)]\}\right\}^{-1}. \tag{12}$$

2. Concentration of the Target Component is High, i.e., $1 - c \ll 1$. Then, in accordance with (1), $a = 1$, $b = -1$, and

$$1 - c' = \frac{\varkappa_0(1-c_0)}{\psi_2}\{\varkappa_i + \exp[-(1+\varkappa_i)\,y_0]\}\{\varkappa_e - \exp[(1-\varkappa_e)(y_e - y)]\}, \tag{13}$$

$$1 - c'' = \frac{\varkappa_0(1-c_0)}{\psi_2}\{\varkappa_i + \exp[-(1+\varkappa_i)\,y]\}\{\varkappa_e - \exp[(1-\varkappa_e)(y_e - y_0)]\}, \tag{14}$$

where ψ_2 is the value of ψ_1 in (8) when $b = -1$.

From the last two formulas we find the degrees of separation in the concentrating and stripping parts of the column:

$$q_e \equiv \frac{1-c_0}{1-c_e} = \frac{\psi_2}{\varkappa_0(\varkappa_e - 1)}\{\varkappa_i + \exp[-(1+\varkappa_i)\,y_0]\}^{-1}, \tag{15}$$

$$q_i = \frac{1-c_i}{1-c_0} = \frac{\varkappa_0}{\psi_2}(\varkappa_i + 1)\{\varkappa_e - \exp[(1-\varkappa_e)(y_e - y_0)]\}. \tag{16}$$

3. Concentration of the Target Component Is in the Range $0.3 < c < 0.7$. Here, as was shown in [2], we must set $b = 0$ in (1). Then (6) and (7) yield

$$c' = c_0 + \frac{a}{\varkappa_0\varkappa_e}\left\{\varkappa_e\,[\exp[-\varkappa_e(y_e - y)] - \exp(-\varkappa_i y_0)] + \varkappa_i\,\{\exp[-\varkappa_e(y_e - y)] - \exp[-\varkappa_e(y_e - y_0)]\}\right\}, \tag{17}$$

$$c'' = c_0 - \frac{a}{\varkappa_0\varkappa_i}\left\{\varkappa_e\,[\exp(-\varkappa_i y) - \exp(-\varkappa_i y_0)] + \varkappa_i\,\{\exp(-\varkappa_i y) - \exp[-\varkappa_e(y_e - y_0)]\}\right\}, \tag{18}$$

and the difference between the concentration is

$$\begin{aligned} c_e - c_0 \equiv (\Delta c)_e &= \frac{a}{\varkappa_e\varkappa_0}\left\{\varkappa_i\,\{1 - \exp[-\varkappa_e(y_e - y_0)]\} + \varkappa_e\,[1 - \exp(-\varkappa_i y_0)]\right\}, \\ c_0 - c_i \equiv (\Delta c)_i &= \frac{a}{\varkappa_i\varkappa_0}\left\{\varkappa_i\,\{1 - \exp[-\varkappa_e(y_e - y_0)]\} + \varkappa_e[1 - \exp(-\varkappa_i y_0)]\right\}. \end{aligned} \tag{19}$$

The cross section at which the mixture being separated should be fed in must be so chosen that at this cross section the concentration inside the column will be equal to the concentration of the input flow, since this eliminates losses in separating power due to the irreversible process of mixing.

From formulas (9), (10), it can be seen that when $y = y_0$, the value of c' and c'' will be equal to c_0 in the case when

$$\varkappa_0\{\varkappa_i - \exp[(1-\varkappa_i)\,y_0]\}\{\varkappa_e + \exp[-(1+\varkappa_e)(y_e - y_0)]\} = \psi_1.$$

Substituting the value of ψ_1 from (8), we obtain a relation determining whether the condition we are interested in, viz.,

$$\varkappa_i\frac{1 - \exp[(1-\varkappa_i)\,y_0]}{\varkappa_i - \exp[(1-\varkappa_i)\,y_0]} = \varkappa_e\frac{1 - \exp[-(1+\varkappa_e)(y_e - y_0)]}{\varkappa_e + \exp[-(1+\varkappa_e)(y_e - y_0)]} = n_1, \tag{20}$$

is satisfied. Analogously, from formulas (13), (14),

$$\varkappa_i\frac{1 - \exp[-(1+\varkappa_i)\,y_0]}{\varkappa_i + \exp[-(1+\varkappa_i)\,y_0]} = \varkappa_e\frac{1 - \exp[(1-\varkappa_e)(y_e - y_0)]}{\varkappa_e - \exp[(1-\varkappa_e)(y_e - y_0)]} = n_2 \tag{21}$$

and from (17), (18),

$$\varkappa_e(y_e - y_0)_{\text{opt}} = \varkappa_i y_{0,\,\text{opt}} \tag{22}$$

It should be noted that in (20), (21) the values of n_1 and n_2 are always less than unity. From these formulas we find that when $c \ll 1$,

$$(y_e - y_0)_{\text{opt}} = \frac{1}{1+\varkappa_e}\ln\frac{n_1+\varkappa_e}{\varkappa_e(1-n_1)},\quad y_{0,\,\text{opt}} = \frac{1}{1-\varkappa_i}\ln\frac{\varkappa_i(1-n_1)}{\varkappa_i - n_1}, \tag{23}$$

and when $1 - c \ll 1$,

$$(y_e - y_0)_{\text{opt}} = \frac{1}{\varkappa_e - 1}\ln\frac{n_2-\varkappa_e}{\varkappa_e(n_2-1)},\quad y_{0,\,\text{opt}} = \frac{1}{1+\varkappa_i}\ln\frac{\varkappa_i+n_2}{\varkappa_i(1-n_2)}. \tag{24}$$

Substituting (23) into (11) and (12) yields the following expressions for the optimum degrees of separation when $c \ll 1$:

$$q_{e,\text{opt}} = \frac{\varkappa_e+n_1}{\varkappa_e},\quad q_{i,\,\text{opt}} = \frac{\varkappa_i}{\varkappa_i-n_1},\quad q_{\text{opt}} = q_{e,\text{opt}}q_{i,\,\text{opt}} = \frac{\varkappa_i(\varkappa_e+n_1)}{\varkappa_e(\varkappa_i-n_1)}. \tag{25}$$

In the case $1 - c \ll 1$, using (13), (14), (24), we obtain

$$q_{e,\,\text{opt}} = \frac{\varkappa_e}{\varkappa_e-n_2},\quad q_{i,\,\text{opt}} = \frac{\varkappa_i+n_2}{\varkappa_i},\quad q_{\text{opt}} = q_{e,\text{opt}}q_{i,\,\text{opt}} = \frac{\varkappa_e(\varkappa_i+n_2)}{\varkappa_i(\varkappa_e-n_2)}. \tag{26}$$

When $c(1 - c) \approx a$, making use of (22), we obtain from (19) the result that

$$\varkappa_i(\Delta c_i)_{\text{opt}} = \varkappa_e(\Delta c_e)_{\text{opt}} = a\left[1-\exp\left(-\frac{\varkappa_i\varkappa_e}{\varkappa_0}y_e\right)\right]. \tag{27}$$

To design the column, we specify the degrees of separation in its concentrating and stripping parts. Then, e.g., in the case $c \ll 1$ the first of relations (25) leads to the formula

$$n_1 = \varkappa_e(q_{e,\,\text{opt}} - 1), \tag{28}$$

which, on being substituted into (23), yields

$$(y_e - y_0)_{\text{opt}} = \frac{1}{1+\varkappa_e}\ln\frac{q_{e,\text{opt}}}{[1-\varkappa_e(q_{e,\text{opt}}-1)]}.$$

Obviously we must have $1 > \varkappa_e(q_{e,\text{opt}} - 1)$, and this leads to the inequality

$$\varkappa_e < \frac{1}{q_{e,\text{opt}}-1}, \tag{29}$$

which determines the upper limit of the dimensionless samples $\varkappa_e$. Starting with inequality (29), we must specify the concrete value of $\varkappa_e$, after which, taking account of (28), we can calculate from the first of the formulas (23) the dimensionless height of the concentrating part of the column. The second of relations (25) enables us, on the basis of the known value of n_1 (see (28)), to determine

$$\varkappa_i = n_1\frac{q_{i,\,\text{opt}}}{q_{i,\,\text{opt}}-1}, \tag{30}$$

and then to determine y_0 from (23).

An analogous approach in the case $c(1 - c) \ll 1$ yields instead of (28)-(30) the result

$$n_2 = (q_{e,\,\text{opt}} - 1)\,\varkappa_e/q_{e,\text{opt}};\ \varkappa_e < q_{e,\,\text{opt}}/(q_{e,\text{opt}} - 1);\ \varkappa_i = n_2(q_{i,\text{opt}} - 1). \tag{31}$$

When $c(1 - c) = a$, we have

$$\varkappa_e < \frac{a}{(\Delta c)_{e,\,\text{opt}}},\ \frac{(\Delta c)_{i,\,\text{opt}}}{(\Delta c)_{e,\,\text{opt}}} = \frac{\varkappa_e}{\varkappa_i},\ y_{0,\text{opt}} = -\frac{1}{\varkappa_i}\ln\left[1-\frac{\varkappa_e}{a}(\Delta c)_{e,\text{opt}}\right]. \tag{32}$$

Making use of formulas (8), (11), (15), (19), (22)-(32) we have constructed a graph (Fig. 2) from which we can conclude that for the given conditions an incorrect arrangement of the input of the initial product may substantially reduce the efficiency of separation. As the ratio q_e/q_i increases (curves 2 and 3), the degree of separation achieved differs little from that achieved in a column with central input. In the case when a substance is being purified, $(1 - c) \ll 1$, for equal values of q_e, q_i the separation efficiency is more sensitive to the placement of the feedpoint of the mixture being separated than in the case of concentration

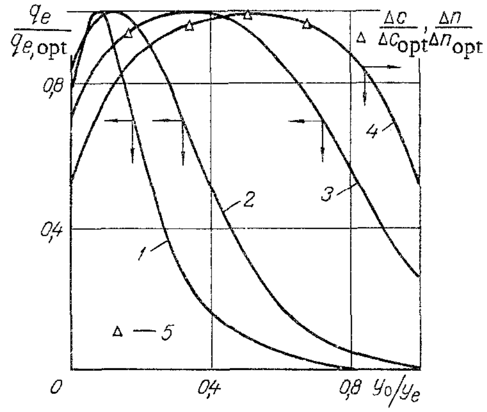

Fig. 2. Variation of $q_e/q_{e,\mathrm{opt}}$, $\Delta c/(\Delta c)_{\mathrm{opt}}$, $\Delta n/(\Delta n)_{\mathrm{opt}}$ as functions of the coordinate of the point of input y_0/y_e: 1) $1 - c \ll 1$: $q_e = 1000$, $q_i = 2$; 2) $c \ll 1$: $q_e = 1000$, $q_i = 2$; 3) $c \ll 1$: $q_e = 1000$, $q_i = 10$; 4) $c(1 - c) = \mathrm{const}$: $(\Delta c)_e = (\Delta c)_i = 0.15$; 5) experimental points.

of a component which is present in low concentrations (curves 1, 2). If the mixture is characterized by an approximately constant value of the product $c(1 - c)$, then on a fairly large segment of the height of the column, $0.3 < y_0/y_e < 0.7$, an arrangement with a central input ensures a degree of separation which is close to optimal.

For the experimental investigation of how the efficiency of separation is affected by the placement of the point of input of the mixture being separated, we used a thermal-diffusion column 1 m high with a gap of 0.6 mm and a temperature difference $\Delta T = 44°\mathrm{C}$. The points of input of the initial product are shown in Fig. 1 by circles. As the mixture being separated, we selected industrial oil I-40A because the ATR-3 apparatus developed at the Institute of Heat and Mass Exchange of the Academy of Sciences of the Belorussian SSR [4] is designed primarily for the fractionation of petroleum products and the determination of optimal conditions for its operation in a continuous-sampling regime at the upper and lower ends of the column, i.e., $\sigma_e = \sigma_i = 2$ ml/h. The separation efficiency was determined from the displacement of the index of refraction, Δn, with respect to that of the initial product. Measurements were made on the IRF-22 refractometer. The results are shown by triangles in Fig. 2. As can be seen, the experimental points have a tendency to lie close to curve 4. This is no accident when we consider that petroleum oils may be regarded, to a certain degree of approximation, as binary mixtures one of whose components is a paraffin hydrocarbon and the other is a cyclic hydrocarbon [5], the two being present in almost equal concentrations. Thus, in the operation of the ATR-3 apparatus in a continuous-sampling regime, it can be recommended that the initial product be introduced near the midpoint along the height of the column.

It is interesting to note another fact which arises out of the above formulas. It is found that the relations obtained for the concentrating part of the column yield the same results as for a column which has optimum energy consumption [6]. Therefore we must reduce somewhat the degree of arbitrariness permissible in the choice of the parameter $\varkappa_e$ determined by the inequalities (29), (31). This parameter should be selected from the optimization conditions given in [6] after which, from (29), (31), we can find n_1 or n_2 and, by the first formulas in (23), (24), find the dimensionless height of the concentrating part of the column.

In conclusion, it should be noted that the results obtained are valid for other countercurrent processes as well. In particular, for a process of delicate separation in rectification the symbols given above may be represented as follows:

$$y = \frac{kzS}{G},\quad y_e = \frac{kLS}{G},\quad y_0 = \frac{kL_0S}{G},\quad \varkappa_e = \frac{P}{G},\quad \varkappa_i = \frac{W}{G},$$

where k is the mass transfer coefficient; z, vertical running coordinate; S, surface per unit of height of the column; L, L_0, total height and the position of the feedpoint, respectively; P, stream of product removed; G, stream of vapor; W, stream of wastes.

NOTATION

τ, transfer of the target component; $y = Hz/K$, H, K, coefficient of the transfer equation $y_e = HL/K$; L, height of the column; $\varkappa_e = \sigma_e/H$, $\varkappa_i = \sigma_i/H$, σ_e, σ_i, samples at the positive and negative ends of the column; z, vertical coordinate; c', c'', c_0, concentrations in the concentrating and stripping parts of the column and in the input flow, respectively; $\sigma_0 = \sigma_e + \sigma_i$; q_e, q_i, degrees of separation in the concentrating and stripping parts of the column; y_0, dimensionless coordinate of the feedpoint of the mixture being separated. Subscripts: opt, optimal; e, i, value of y at the positive and negative ends of the column.

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EMISSION OF AN EXPANDING PLASMOID

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The expansion of a radiating plasmoid into a vacuum is examined. It is shown that with an external energy input of $\sim 10^{12}$ W/cm^2 the bulk of the incident energy flux is lost as radiation.

In this paper we attempt an analytical investigation of the steady-state expansion of a plasmoid into a vacuum [1] with due regard to the loss from the plasma due to radiation. A relation between the emitted power and the plasma parameters is found. The dynamics of expansion of the plasmoid is considered. Approximate equations for estimation of the radiative loss and the plasma parameters in the case of a power-law time dependence of the energy input are obtained. Calculations show that the emission can greatly affect the expansion dynamics. A comparison is made with known experiments on the interaction of high-current electron beams with a target.

Emission of Plasmoid. We consider the bremsstrahlung, recombination, and line emission of the plasma. The assessment of reabsorption in the plasma is rather difficult and requires consideration of the contribution of many levels for different ions. In a rough approximation, we confine ourselves to estimates for an average ion $k \sim k_{av}$, and we will ignore reabsorption of bremsstrahlung and recombination radiation in the target plasma.

Bremsstrahlung. We evaluate the emission intensity from the equation [2]

$$Q_e^{\text{brem}} = 1.54 \cdot 10^{-25} N_e T_e^{1/2} k_{\text{eff}} N \ [\text{erg/cm}^3 \cdot \text{sec}].$$

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